Mathematical modeling of emission in small-size cathode

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We consider mathematical modeling of heat transfer and melting through emission in a small-size truncated conical semiconductor cathode.

Our model consists of heat equation (with right-hand side related to Joul heat), equation for the current density inside the cathode and equation relating the current density to the electric potential, and conditions on the free boundaries (boundaries between the solid and liquid phases) which are called Stephan and Gibbs-Tomson conditions.

Our model must take into account some specific features of real cathode such as small size (height of the cathode is about 10 – 15 um) and very high current density which is a reason of possible melting. The latter implies a free boundary problem.

Though all equations are linear, the whole problem becomes nonlinear because of the presence of free boundaries whose location is unknown and their finding is also a part of problem. We use the following technique: we consider this problem as the limiting (in the weak sense) problem for the other problem known as the phase-field system:

$$\frac{\partial\theta}{\partial t}+\frac{l}{2}\frac{\partial\varphi}{\partial t}=k\Delta\theta,\ \ \varepsilon^{2}\frac{\partial\varphi}{\partial t}=\varepsilon^{2}\Delta\varphi+\varphi-\varphi^{3}+\varepsilon\kappa\theta.$$

Here $\varepsilon\to 0$ is a small parameter and in the limit we obtain the model described above.

Though our model assumes a situation where the cathode has a domain occupied by the liquid phase, it cannot be used in situations where the liquid phase domain appears in the initially solid cathode. So we propose our own method for incorporating the nuclei, which is an external method with respect to the phase field system. We incorporate a nucleus as an unstable state in a small volume of the old phase, or we can say that we artificially create an unstable ``mushy region" in a small volume of the old phase. Of course, this raises the problem under what conditions a ``mushy region" can be created.

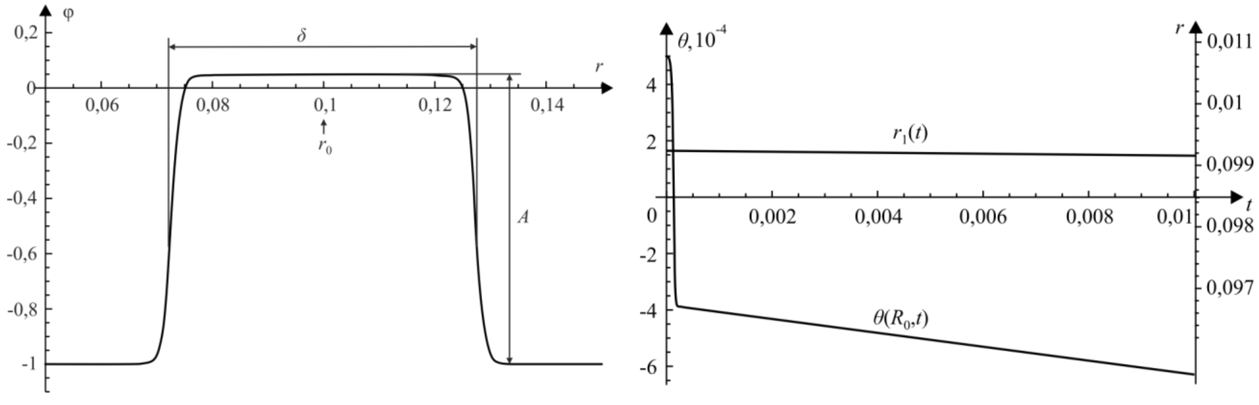
As the nuclei we consider the family of functions which are sums of simple waves for $t=0$:

$$\varphi=A-(A+1)/\left(1+\exp\left(-\frac{r-(r_0+\delta/2)}{\beta\varepsilon\sqrt{2}}\right)\right)-(A+1)/\left(1+\exp\left(\frac{r-(r_0-\delta/2)}{\beta\varepsilon\sqrt{2}}\right)\right).$$

Typical nucleus is shown in the figure below.

Now we present our algorithm for the nucleus incorporation. We calculate the temperature till the time moment at which it begins to exceed the melting temperature. At this moment, the computation stops (the process modeling is stopped), and then it starts again with the calculated temperature taken as the initial condition for θ and with φ given by the nucleus equation. We also take into account the possible increase in the Nottingham effect after liquid phase formation.

The plot below shows position of the left free boundary of liquid phase domain and temperature in the top surface of cathode in dimensionless units as $\theta=0$ is the melting temperature. We can see that after the liquid phase formation, the temperature begins to decrease and the left boundary moves to the right. After some time, the liquid phase must disappear, and the temperature will increase again. This brings the system to the periodical mode of sequential melting-solidification. But because of our restricted calculation resources, we cannot model the whole situation.



Plots of the liquid phase nucleus and time-dependence of the temperature and the free boundary position.