RESONANCE PHENOMENA IN EXTENDED MATHIEU EQUATION: THEORY AND SIMULATION

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Phenomenon and mechanisms of an energy transfer between degrees of freedom of a dusty plasma system are of great interest in the field of dusty plasma [1,2]. One of such mechanisms [1] is based on parametric resonance. Such phenomena can be described by the extended Methieu equation [1,3]: $\ddot{x}+2\lambda\dot{x}+\omega_0^2\left(1+h\cos\omega_p t\right)x=\eta(t)$, where λ is a friction coefficient, h is an amplitude of modulation, ω_0 is an eigen frequency of system and ω_p is a frequency of parameter, $\eta(t)$ is a stochastic force with zero mean value.

Classical Mathieu equation is studied for $h \ll 1$, $\lambda = 0$ and $\varepsilon = \omega_p - 2\omega_0/n \ll \omega_0$ on the level of the first order of accuracy [4].

Acting by an analogy with [4] and using averaging over an ensemble of distributions of $\eta(t)$, we can obtain an expression for the growth rate of the amplitude s. Since this approach can be used only for small values of parameters, while there may be significant friction force and h may be above 1, parallel computer simulation is used to obtain numerical solution for a wide range of parameter values. The resonance areas boundaries obtained analytically and the ones obtained numerically are close only for non-friction system with $h<1$.

The approach proposed in [4] leads to serious differences with the numerical solution of the equation in the presence of friction ($\lambda \neq 0$).This is because this approach takes into account only terms of zero-order of accuracy with λ. So this approach works only with assumption that $\lambda \ll 1$ and for solution of the first order of accuracy. The authors have proposed a solution that takes into account terms of other orders of smallness. The results obtained this way are closer to the data obtained numerically. It also explains such phenomenon as the shift of the point with a minimum value of h in which the resonance occurs.

The extended Mathieu equation is studied by analytical approach and computer simulation. The solutions for higher order of accuracy are obtained. The method for numerical solution and for estimation of the resonance zone, the time of onset and the growth rate of the amplitude is proposed. The solution for the extended Mathieu equation is obtained for wide range of parameter values. The results of numerical solution are compared with analytical solutions of different order and known analytical results for Mathieu equation. The theory is improved in order to reduce inaccuracies.

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